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Inverse Simulation of Aeroassisted Orbit Plane Change of a Spacecraft

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The orbit plane change of a spacecraft, which is able to perform synergistic maneuvers, is dealt with as an inverse problem, extending to aerospace vehicles' definitions and themes of recent aircraft flight mechanics developments. A few single significant missions of increasing complexity are first considered, and the necessary related control actions are evaluated, taking into account the full set of motion equations. Then complete aeroassisted optimal trajectories of the vehicle as a point mass are carried out, and the method of inverse simulation is successively used to evaluate the control actions for the center of gravity and the attitude motions. Comparisons with propulsively controlled missions are presented, and the advantages of synergistic maneuvers are shown.

Nomenclature

C_h = heat-transfer coefficient
C_{mx}, C_{my}, C_{mz} = aerodynamic moment coefficients in body axes
C_p = pressure coefficient
C_t = shear stress coefficient
C_x, C_y, C_z = aerodynamic force coefficients in body axes
D = aerodynamic drag, N
E = longitude, deg
g = acceleration of gravity, m s^{-2}
h = altitude, m
Kn = Knudsen number
L = aerodynamic lift, N
l = reference length, m
M = Mach number
m = mass, kg
N = latitude, deg
n_c = number of controls
n_s = number of desidered output
P, Q, R = component of relative angular velocity in body axes, rad s^{-1}
p, q, r = component of angular velocity in body axes, rad s^{-1}
R_g = geocentric radius, m
T = thrust, N
V = speed, m s^{-1}
α, β = angle of attack and sideslip, respectively, deg
γ = flight-path angle, deg
Δt = time duration of thrust, s
δ = angle of control, deg
δ_T = thrust level T/T_{MAX}
η = local incidence, deg
λ = mean free path of molecules, m
ν_T = frequency of burning, s^{-1}
σ = bank angle, deg
χ, θ, ψ = Euler's angles, deg
ψ_v = heading angle, deg
ω = Earth angular velocity, rad s^{-1}

Subscripts

a = aileron
DES = desired output
e = elevator
f = final value
i = initial value
opt = optimal value
p = propellant
r = rudder

Introduction

THE possibility of improving the performance of spacecraft by adopting both aerodynamic and propulsive controls has been considered in a number of works in cases where the aerospace vehicles are able to maneuver in a sufficiently dense atmosphere.[1–3] In this framework one of the subjects that has received great attention is the problem of the orbit plane change because the fuel consumption to realize such a mission is very significant even for small angle variations.[4] The opportunity of adopting aerodynamic control surfaces is subjected to many constraints and limitations among which the necessity of sustaining high temperatures and heat-transfer rates at speeds close to the orbital ones and at altitudes where the aerodynamic pressures are significant.

One of the consequences of what we just observed is the fact that dealing with the aeroassisted orbit plane change is by necessity a search for optimal solutions as far as the trajectory, the spacecraft angle of attack, the thrust level, and the consumption of propellant; all functions of the time are considered. On the other hand, once a desirable trajectory of the c.g. has been assigned or optimum laws have been evaluated for c.g. and attitude motions, then one faces the problem of evaluating the proper actions of the aerodynamic and thrust controls. This last problem represents an inverse flight problem, and its solution for some basic and optimal maneuvers of a variable mass spacecraft is the main contribution of this work.

We note that solving the full set of equations represents a cumbersome computational task because the timescales for the c.g. motions and for the attitude variations can differ by orders of magnitude. In particular, the attitude variations following the pertinent control surface actions are much faster than the effects of the aerodynamic and thrust controls on the c.g. motion.[5] As a consequence, the basic equations governing the c.g. motion and the attitude can be conveniently solved separately. In particular, this is true under the hypothesis that the aerodynamic force coefficients are independent of the control angles.

Apart from possible computational instabilities when the full set of equations is solved at the same time, great savings of computer's time are obtained by timescale separation when dealing with optimization procedures, as is the case of the inverse problem solving. In fact, in this circumstance a time consuming, local optimization routine is used at each time step when the control actions are calculated as we recall in Appendix B, devoted to inverse problem solving.

At this point we introduce the main subject of the paper by dealing with the inverse problems of an aerospacecraft. Research on these problems both in airplane and helicopter dynamics is fast developing after the first pioneer work by Kato and Sugiura.[6] Here we will

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extend some definitions of the aircraft inverse problems[7] to those of an aerospace vehicle with aerodynamic controls and will recall and successfully apply a solution technique that is very effective in dealing with airplane dynamics.

Let $\boldsymbol{x} = (V, \alpha, \beta, \chi, \theta, \psi, p, q, r, N, E, R, m)$ be the spacecraft state,[8] where m appears in addition to the state variables of the classic atmospheric flight mechanics of aircraft. We say that an inverse problem is nominal when the number of controls n_c is equal to the number n_s of assigned scalar output variables $\boldsymbol{y}$, which are connected with $\boldsymbol{x}$.

For $n_c = n_s$ the solution, if it exists and is unique, corresponds to the solution of an algebraic set of equations. For $n_c - n_s = n_r \geq 1$ the problem is n_r times redundant, and the search for solutions is associated with a number of suitable arbitrary constraints.[7]

We will first give the characteristics of a reference spacecraft that will be considered in the applications and recall the way for evaluating the aerodynamic forces and moments. Subsequently, the main aspects of the inverse simulation dynamics will be presented and followed by an analysis of three inverse problems. The first two of them are nominal, and the third one is redundant, and they deal with simple maneuvers where the aerodynamic effects can play a fundamental role.

The last treated problem corresponds to the orbit plane change of minimum fuel consumption. To this question several solutions are known that concern the mass point trajectory as controlled only by propulsive actions. Synergistic maneuvers have been more recently dealt with for simple arcs of the trajectory and, again, within the point mass approximation and without consideration to the control actions. Here the optimal aeroassisted orbit plane change is calculated as a whole, and then the controls are evaluated by inverse simulation.

Spacecraft

In all that follows and just for reference purposes, we will consider an aerospace vehicle like the one that is sketched in Fig. 1, where its main characteristics are also reported. To evaluate the aerodynamic actions on this spacecraft in that region of the atmosphere between about 70 and 80 km of altitude, where it is proposed in the literature that the vehicle can conveniently operate, semi-empirical expressions will be used.[9] These expressions proved to be in very good agreement with the results of more complicated theories[10] and were validated against accurate experimental data.[11] With respect either to the computer's time-consuming numerical solution of the Boltzmann equation models or to the direct simulation Monte Carlo results, these expressions are thought to be accurate enough and speedy in calculating the aerothermodynamic coefficients. In the present applications, in particular, for a flight Knudsen number Kn falling in the transition flow regime, the pressure, shear stress, and heat-transfer coefficients are given by

$$C_J(\eta, M, Kn) = C_{J,\mathrm{FM}}(\eta, M) - [C_{J,c}(\eta, M) - C_{J,\mathrm{FM}}(\eta, M)] \exp\left(-1/\sqrt{2}Kn\right)$$

where $J = p, t, h$; η is the local incidence angle of the stream on an elemental surface; M is the flight Mach number; and $Kn = \lambda/l$, the vehicle overall length. The index FM stays for the value in free molecular flow (see Appendix A), and c stays for continuum flow.

Table 1 shows the most important aerodynamic data for our vehicle at indicative values of M and Kn. In particular, $Kn = 0.07$ corresponds to an altitude of about 80 km. The moment coefficients are referred to the c.g. The control angles δ_e, δ_a, δ_r are calculated under the assumption that the aerodynamic control surfaces, i.e., elevator, ailerons, and rudder, are about 8% of the reference lifting surface of 26.62 m^2. In the computations the maximum value of the heat-transfer rate, calculated according to the preceding expression, had to be less than 50 W cm^{-2} and the maximum local temperature level less than 1666 K, as is usually assumed.[12]

The solutions to the problems that will be carried out can be more or less significantly affected by the choice of a vehicle configuration as the one considered here. This fact is particularly true when one deals with the question of the optimal change of the orbit plane angle. In this respect, one should consider as an open problem the optimal configuration of a spacecraft designed for synergistic maneuvers, where an accurate evaluation of the empty weight of an aerodynamically controlled spacecraft should be carried out and the results compared with the weight of a conventional one, after a complete mission has been assigned. However such an optimal design is not the goal of this work, which is aimed at introducing the inverse simulation as a powerful tool for studying the dynamics and control also of unconventional space vehicles.

As far as the thrust is concerned, we will assume throughout the paper a specific impulse of 400 s. In the cases that will be discussed later, we will assume either the possibility of a continuous variation of the thrust level δ_T or the use of on–off pulses of different time lengths Δt at a constant level T_0 with frequency ν_T. The atmosphere model is the one reported in Ref. 13.

Analysis of Inverse Maneuvers

A summary of the foundations of the inverse simulation dynamics and a recall of its solution method are given in Appendix B. In this section we introduce a few preliminary simulations of interesting inverse problems to show the potential of the method and the analogies with the conventional aircraft cases before carrying out the all important search for the optimal fuel-consumption problem. To appreciate the method, we will proceed by steps, from a simple maneuver to more complex ones, and will first present the nominal two-dimensional case 1 of an orbit radius increase. Then the three-dimensional case 2 of an assigned latitude variation with the time will be dealt with where the four controls, namely δ_e, δ_a, δ_r, and either δ_T or ν_T, intervene. An imposed law for the change of the orbit plane angle is finally considered in the redundant situation of case 3 to conclude the section and to introduce the subject of the optimal maneuver. As already stated, whereas the solution to the nominal cases is obtained by an algebraic algorithm, the redundant cases need n_r further conditions or constraints be added to the n_s imposed outputs. These constraints can be expressed in various ways and, in particular, by imposing that some minimum conditions be locally satisfied by functions of the state and control variables.

The full set of motion equations for the c.g. and for the spacecraft attitude can be easily found in textbooks of Refs. 1 and 8 and is reported in Appendix C.

1) The first case involves a maneuver in the equatorial plane. The spacecraft is initially at the equilibrium speed $V_i = 7315$ m s^{-1} at an altitude $h_i = 75$ km under the action of the centrifugal and aerodynamic forces and of gravity. Assigned variations of the two variables v and h are imposed so that, at the end, the vehicle achieves the speed $V_f = 7460$ m s^{-1} at $h_f = 80$ km:

$$V_{\mathrm{DES}}(t) = V_i + (V_f - V_i) f(t)$$
$$h_{\mathrm{DES}}(t) = h_i + (h_f - h_i) f(t) \tag{1}$$

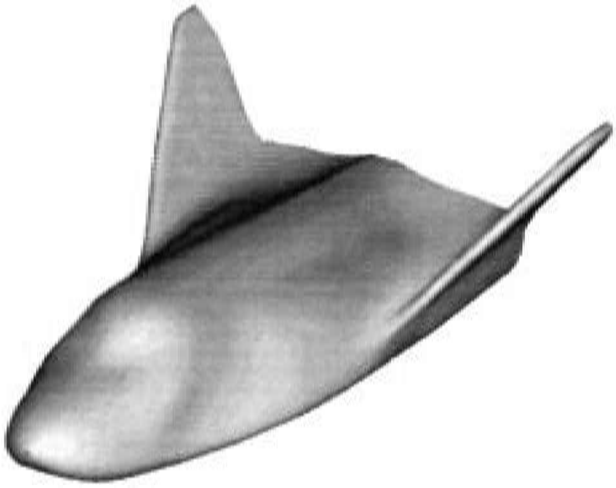

Fig. 1 Reference spacecraft. Total mass m_i = 8562 kg; inertia moment I_{xx} = 10,185 kg m^2; inertia moment I_{yy} = 45,547 kg m^2; inertia moment I_{zz} = 48,326 kg m^2; reference area S = 26.62 m^2; overall length L = 8.62 m; and wing span b = 4.23 m.

Table 1 Aerodynamic coefficients of the spacecraft ($M = 21$, $Kn = 0.07$)

	β, deg					
α, deg	0	5	10	0	5	10
		C_x			$C_{mx} \times 10$	
−45	−0.103	−0.103	−0.103	0.000	0.114	−0.161
−35	−0.097	−0.097	−0.098	0.000	0.226	0.181
−25	−0.087	−0.087	−0.088	0.000	0.348	0.518
−15	−0.074	−0.075	−0.076	0.000	0.471	0.813
−10	−0.067	−0.068	−0.069	0.000	0.521	0.894
−5	−0.060	−0.061	−0.063	0.000	0.566	0.844
0	−0.055	−0.056	−0.058	0.000	0.529	0.537
5	−0.051	−0.052	−0.053	0.000	0.166	−0.170
10	−0.048	−0.049	−0.051	0.000	−0.535	−1.267
15	−0.047	−0.048	−0.049	0.000	−1.358	−2.685
25	−0.048	−0.048	−0.049	0.000	−2.877	−5.686
35	−0.050	−0.050	−0.051	0.000	−4.153	−8.348
45	−0.052	−0.052	−0.053	0.000	−5.213	−10.570
		C_y			$C_{my} \times 10^3$	
−45	0.000	−0.072	−0.142	1.091	1.087	1.077
−35	0.000	−0.063	−0.126	0.690	0.689	0.687
−25	0.000	−0.053	−0.106	0.356	0.358	0.364
−15	0.000	−0.042	−0.083	0.134	0.138	0.148
−10	0.000	−0.036	−0.072	0.077	0.081	0.090
−5	0.000	−0.030	−0.063	0.057	0.059	0.063
0	0.000	−0.027	−0.056	0.065	0.064	0.059
5	0.000	−0.024	−0.052	0.079	0.072	0.057
10	0.000	−0.024	−0.050	0.066	0.057	0.034
15	0.000	−0.025	−0.050	0.019	0.010	−0.014
25	0.000	−0.027	−0.054	−0.177	−0.184	−0.203
35	0.000	−0.030	−0.060	−0.485	−0.488	−0.499
45	0.000	−0.033	−0.066	−0.863	−0.863	−0.865
		C_z			$C_{mz} \times 10^3$	
−45	0.638	0.637	0.633	0.000	5.124	10.200
−35	0.454	0.454	0.454	0.000	3.715	7.475
−25	0.284	0.285	0.289	0.000	2.194	4.557
−15	0.147	0.149	0.156	0.000	0.640	1.606
−10	0.096	0.099	0.106	0.000	−0.103	0.249
−5	0.058	0.060	0.066	0.000	−0.687	−0.641
0	0.028	0.029	0.033	0.000	−0.882	−0.733
5	−0.007	−0.006	−0.004	0.000	−0.475	−0.010
10	−0.055	−0.055	−0.054	0.000	0.491	1.433
15	−0.119	−0.118	−0.117	0.000	1.665	3.410
25	−0.289	−0.288	−0.285	0.000	3.927	7.727
35	−0.504	−0.501	−0.494	0.000	5.910	11.620
45	−0.741	−0.737	−0.724	0.000	7.594	14.960

where $f(t) = t/t^* - 1/(2\pi)\sin(2\pi t/t^*)$, with $t^* = 500$ s. The problem is nominal because either the throttle level δ_T or v_T for $T_0 = 89{,}000$ N and the elevator angle are the two possible controls.

The main results are shown in Fig. 2, where both the on–off and the continuous case of thrust variation are reported. It is possible to realize that in both cases the dependence of the two control characteristics upon the time is the same. Consequently, for the case that uses the on–off control, the consumption of propellant m_p is practically the same as in the case where δ_T changes. Note the decreased value of the angle of attack (referred to the x-body axis) caused by decreased weight of the spacecraft at the end of the maneuver.

2) A three-dimensional nominal maneuver is considered next. The spacecraft performs a variation of its latitude as an assigned function of the time while keeping its initial speed and altitude. A last condition is imposed to keep the sideslip under control:

$$N_{\mathrm{DES}}(t) = N_{\max}\sin^3(t/\pi t^*), \qquad V_{\mathrm{DES}} = V_i$$
$$h_{\mathrm{DES}} = h_i, \qquad p_{\mathrm{DES}} = -\chi_0\chi + \chi_1\beta \tag{2}$$

with $N_{\max} = 1$ deg, $t^* = 2000$ s, $V_i = 7160$ m s^{-1} at $h_i = 75$ km, $\chi_0 = 2\sqrt{[\chi_1(g/V - V/R)]}$, and $\chi_1 = 30$ s^{-1}. This condition imposed on p is equivalent to assigning an artificial dihedral to the vehicle. The controls are δ_T, δ_e, δ_a, δ_r. Figure 3 shows the main results, and only $\delta_T(t)$ is reported as far as the thrust is concerned. Again, the final value of δ_e is different from the initial one as the mass has changed of about 18% during the maneuver. Note the Δi variations with the time. δ_a undergoes extremely small variations with respect to the other two aerodynamic controls.

3) A final case is now discussed that corresponds to assigning a variation with the time of the inclination angle of the orbit plane according to the law $\Delta i/\Delta i_f = f(t)$, where $f(t)$ is given again by the expression just reported but with $t^* = 1000$ s and $\Delta i_f = 10$ deg. The velocity and the altitude are to be kept constant: $V_i = 7160$ m s^{-1} and $h_i = 75$ km.

In this situation the problem is still nominal if we again impose to keep the sideslip under control; otherwise, it becomes redundant, with $n_c - n_s = 1$, when β is left to change. Imposing the sideslip condition is usual in aircraft dynamics to prevent uncomfortable lateral accelerations. In space applications to an unmanned vehicle, the sideslip prevention condition can be relaxed in favor of a condition which imposes that the cost function $J(\boldsymbol{x}, \boldsymbol{u}) = \delta_r^2 + \delta_a^2 + \beta^2$ be locally minimum. This last constraint is considered in order to penalize either the asymmetrical controls and the sideslip, which usually involve high drag and fuel consumption. The results are shown in Fig. 4. Appreciable differences between the nominal and the redundant cases are evident as far as $\delta_a(t)$ and $\delta_r(t)$ are concerned. The main control actions however pertain to the thrust and to the elevator, and they are practically the same in both cases. Furthermore, the difference of consumptions is almost negligible, and the final m_p/m_i is a little more than 22%. As we will see, this consumption is very close to the one for an optimized trajectory.

Our attention will now be addressed to the important question of calculating the optimal solution to the aeroassisted change of the angle of the spacecraft orbit plane. This problem has been already treated in the past in the framework of simplified models that essentially considered optimum c.g. trajectories with limit values to

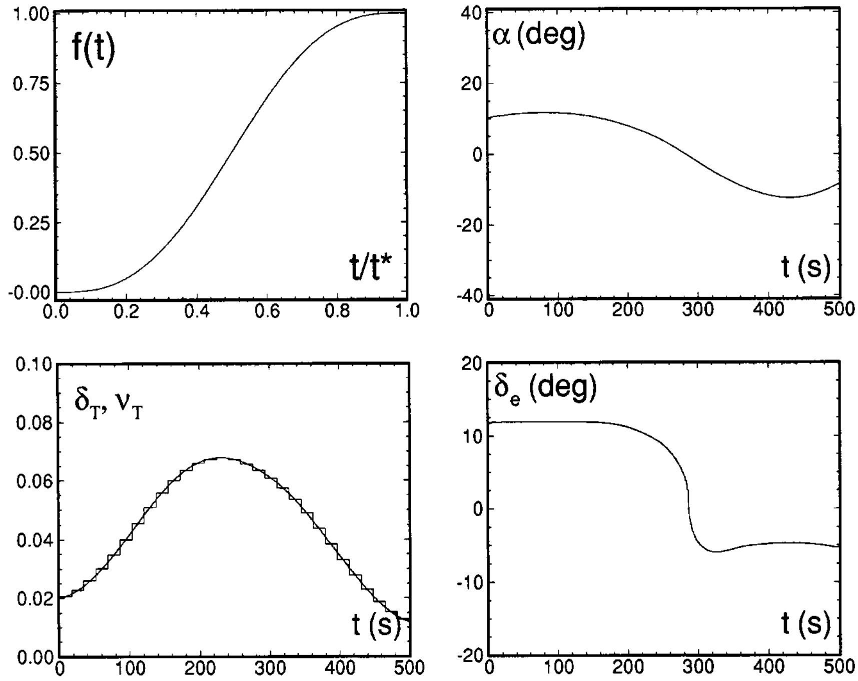

Fig. 2 Solution for an assigned change of the orbital radius.

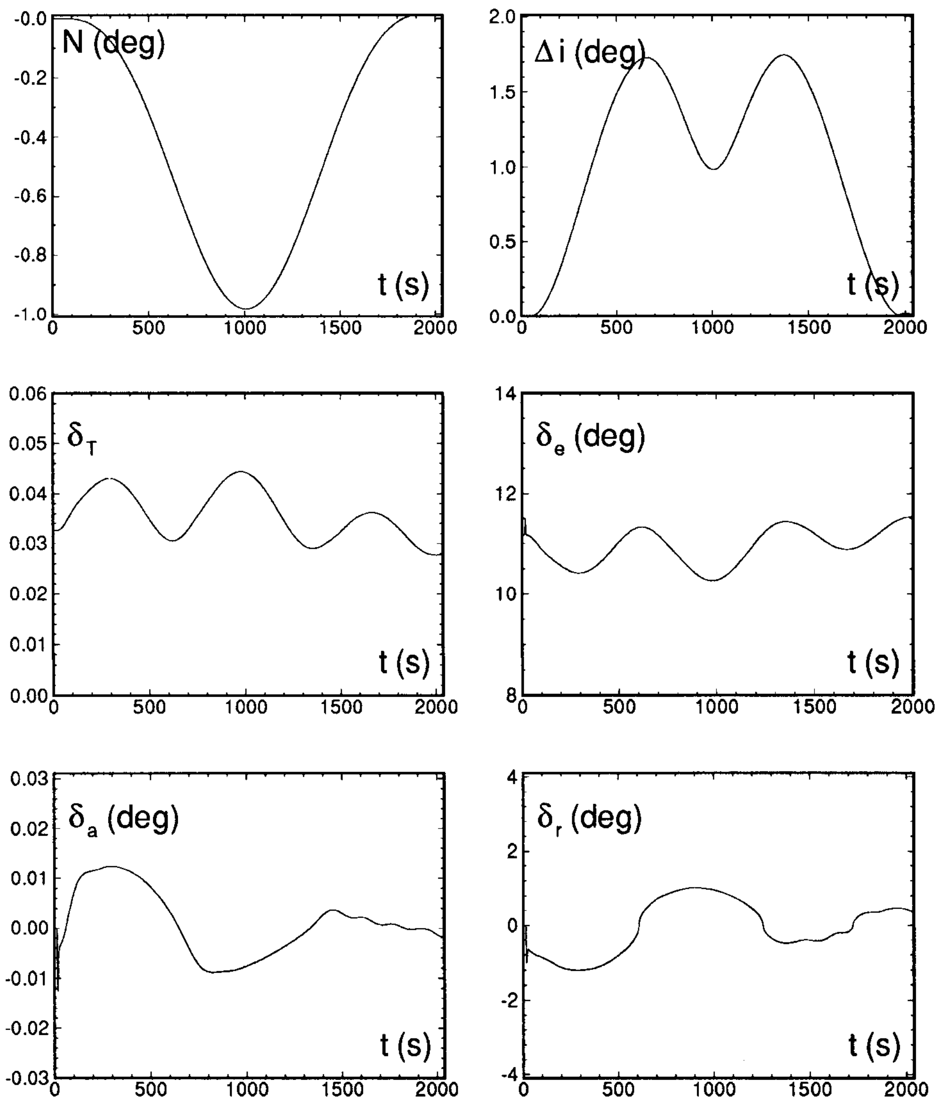

Fig. 3 Solution for an assigned change of latitude.

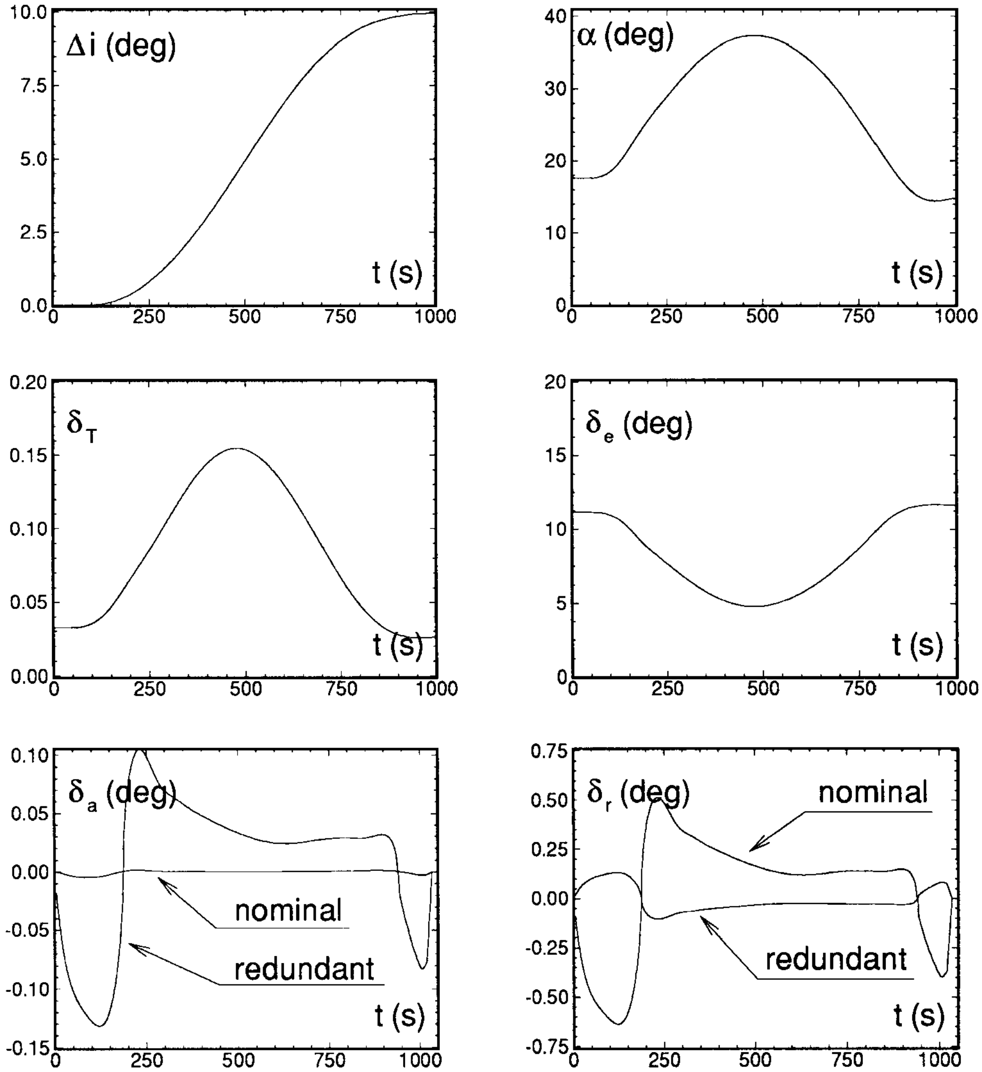

Fig. 4 Solution for a change of the orbit plane angle $\Delta i_f = 10$ deg.

either the heat-transfer rate and/or the temperature at the wall of the vehicle.[1,2] With this in mind, the main results of the pertinent papers concerned the value of the angle of attack, which is necessary to obtain the best performance.

Here we again deal with the optimum c.g. trajectory for minimum fuel consumption and within the heating limits, but the question is treated by solving first the full set of motion equations for the c.g. and, subsequently, by evaluating, the set of control actions such that an optimum trajectory can be actually realized.

Optimal Change of the Orbit Plane Angle

A number of works are present in the literature concerning the aeroassisted optimal orbit transfer. In some of them, the search for an optimal mission is based on a division of the entire trajectory in characteristic arcs that are singularly optimized. As a consequence, the results lose in part their generality. Here the entire c.g. trajectory is considered as a single maneuver and the optimal solutions found on a minimum fuel-consumption basis. At the end we can still recognize peculiar characteristics to some arcs of the trajectory that somehow validate previously obtained results. As a reference and for a better understanding of the mission, we cite here the main aspects of the most significant maneuvers as some of them were singularly considered in the literature.[14,15]

Deorbiting

The aerospace vehicle is initially in equilibrium on a circular orbit around the Earth. As a consequence of an impulsive change of its velocity caused by a thrust impulse, the spacecraft decreases its altitude in the orbit plane and in the absence of aerodynamic forces.

Aeroturning

At an altitude where the aerodynamic effects are sizeable, acting on the combined aerodynamic and propulsive controls causes the desired change of the orbit plane angle. The duration of this phase, and the time laws for thrust, angle of attack, and bank angle are unknown. The state of the spacecraft is governed by the c.g. motion equations. The heat-transfer rate must be sustainable.

Reorbiting

This is the ascent phase, which would be at zero cost in the absence of aerodynamic forces. The vehicle goes back to the initial altitude.

Circularization

A thrust impulse is applied as the spacecraft enters into a new circular orbit in a different plane but of the same radius of the original one.

As said, we carried out the solution by treating the problem as a whole, i.e., after assigning the initial and the final conditions and the desired Δi_f, we implemented the c.g. trajectory optimization procedure and then solved the control problem without any previous division of the trajectory into tracts. We assumed that the initial deorbiting and the final circularization were obtained by unknown single thrust impulses, while a limited heat-transfer rate and a limited maximum temperature were the constraints.

The optimum problem for the c.g. trajectory was solved by means of a numerical code,[16] which proved to be fast and accurate for our purposes. The quantities α_{opt}, σ_{opt}, and T_{opt} as functions of time represented the unknown variables (pseudocontrols) together with the initial and final thrust impulses.

At this point we consider the redundant inverse problem of evaluating the control laws that allow the desidered optimal c.g. performances. In particular, one assigns $\alpha_{\mathrm{opt}}(t)$, $\sigma_{\mathrm{opt}}(t)$, $V_{\mathrm{opt}}(t)$ with the four controls ν_T or δ_T, and δ_e, δ_a, δ_r and the one-time redundant problem is solved by imposing that the cost function $J(\boldsymbol{x}, \boldsymbol{u}) = \delta_r^2 + \delta_a^2 + \beta^2$ be locally minimum. The basic set of equations includes the moment and control equations, which are to be dealt with in a way similar to the aircraft case.[7]

Figures 5a and 5b present the results of some calculations, which are in any case relative to h_i equal to 180 km, with an initial velocity equal to the orbital one, $V_i = 7322$ m s^{-1}.

The calculated control actions are given here as functions of t for two cases of optimal trajectories corresponding to $\Delta\ i_f = 5$ and 10 deg, respectively.

In the figures the deorbiting and reorbiting phases can be recognized. These phases largely correspond to tracts of Hohmann trajectories at zero lift and negligible aerodynamic drag. Also recognizable is the aeroturning, which begins as soon as the synergistic effects can take place at an altitude of about 90 km. From that altitude changes of the control angles are followed by sizeable variations of the lift and $\Delta\ i$ changes. As the requested $\Delta\ i_f$ is being achieved, negligible drag values are calculated, and a second tract of a Hohmann trajectory takes the spacecraft back to the initial orbit where a thrust impulse provides for the final circularization.

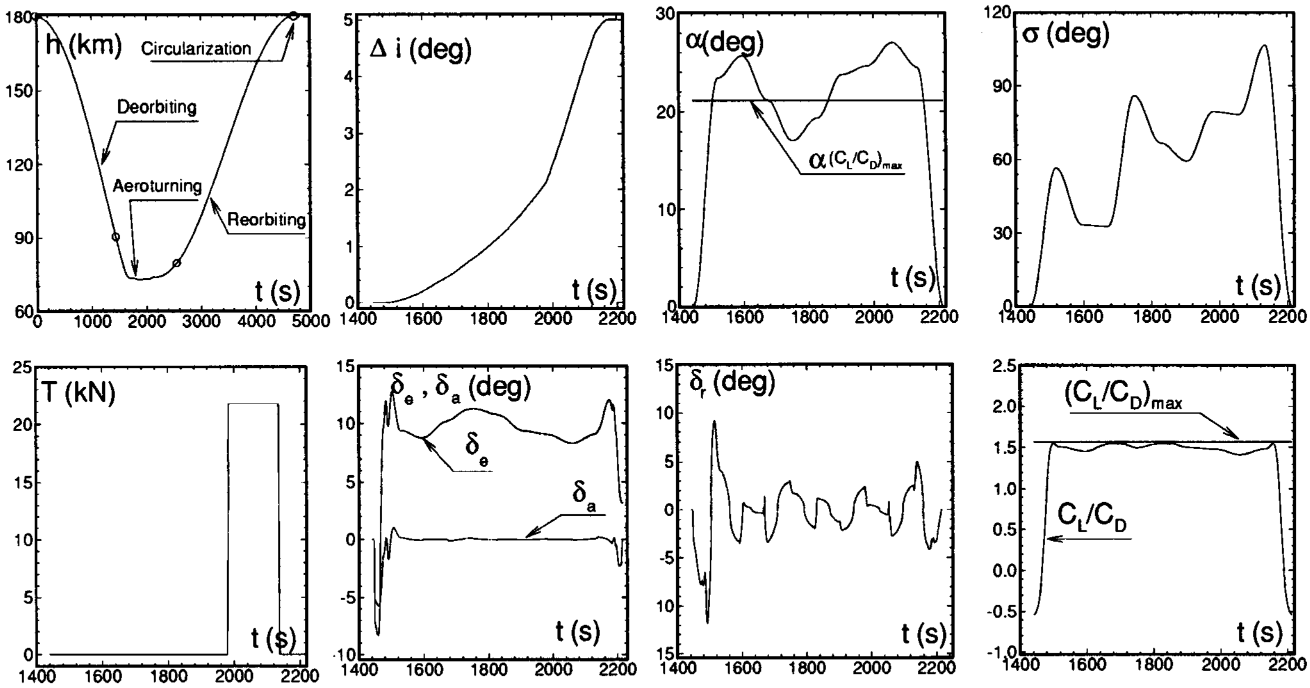

Fig. 5a Optimal trajectory for $\Delta i_f = 5$ deg. From the left: h, Δi, α, σ, T vs t. Necessary controls $\delta_e(t)$, $\delta_a(t)$, $\delta_r(t)$, and efficiency vs t.

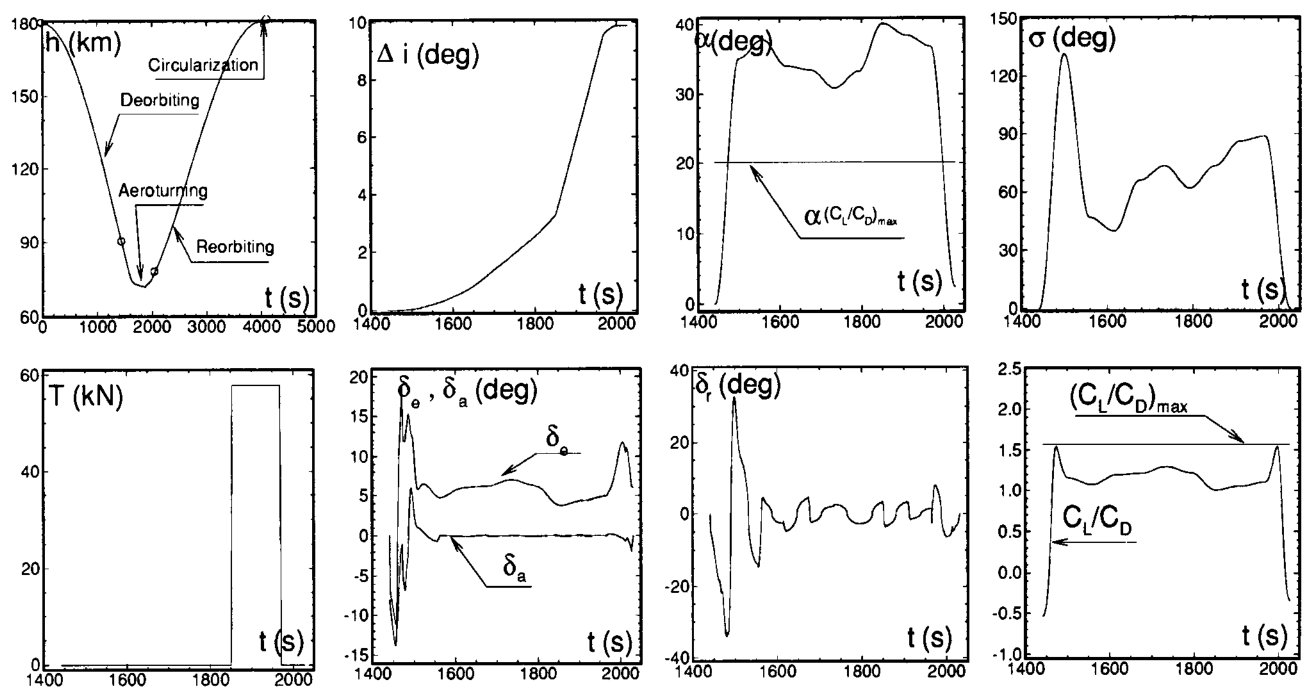

Fig. 5b Optimal trajectory for $\Delta i_f = 10$ deg. From the left: h, Δi, α, σ, T vs t. Necessary controls $\delta_e(t)$, $\delta_a(t)$, $\delta_r(t)$, and efficiency vs t.

We cite here that the classical condition for maximum performance, as stated in simplified theories, i.e., to fly at maximum flight efficiency, is not achieved all over our calculated maneuvers. This result may be in part explained by observing that, in the classic situation, the tracts into which the entire trajectory is ideally divided are thought to be run in steady-state conditions. Further causes can be found in the limiting value of the maximum heat-transfer rate and above all in the fact that treating the problem as a whole problem means that at the end of the aeroturning not only the required Δi_f must be achieved but also the spacecraft state has to be such that the final condition of returning to the prescribed orbit be satisfied.

Our reference spacecraft has a maximum aerodynamic efficiency $(C_L/C_D)_{\max}$, which is a function of M, Kn, and which, therefore, changes with the time along the trajectory. Also recall that the zero-lift angle of attack is a function of M and Kn.

The figures show α and C_L/C_D vs t together with the maximum possible efficiency and the corresponding angle of attack in the same points of the trajectory. The values of $(C_L/C_D)_{\max}$ and $\alpha[(C_L/C_D)_{\max}]$ do not practically change in the considered synergistic phase. In the figures the thrust variations are not represented. We report here that in the two cases $\Delta i_f = 5$ and 10 deg thrust values of $T = 21.8$ kN from $t = 1984$ to 2131 s and $T = 58$ kN from $t = 1853$ to 1968 s, respectively, were calculated.

Conclusion

In our calculation we did not consider all of the constructive details of a spacecraft for synergistic maneuvers. This fact left a degree of uncertainty from the point of view of the weights associated with a realistic configuration of the control system. Even so a comparison of the mass consumptions relative to a conventional and a synergistic maneuver is thought to be meaningful and indicative of the possible advantages of aerodynamic controls. With this in mind the aeroassisted optimal plane change values of $m_p/m_i = 0.12$ for $\Delta i_f = 5$ deg and 0.22 for 10 deg favorably compare with the corresponding 0.17 and 0.30 in case of orbit plane changes, which are obtained by propulsive means only.[1,2]

We conclude the paper by observing that the treatment of synergistic maneuvers can be conveniently approached as an inverse problem by adopting techniques that have been developed for aircraft inverse simulations during the last two decades and are the subject of active research. Along this line it seems that other problems related to optimal performances of aerospace vehicles can be carried out as they represent a field of noticeable interest for future investigations.

Appendix A: Aerodynamic Coefficients in Free Molecular Flow

Here the expressions of the aerothermodynamic coefficients obtained in free molecular flow as functions of the Mach number are reported[9]:

$$
\begin{aligned}
C_{p,\mathrm{FM}}(\eta; M) &= 1/c^2\Big\{\Big[(2-\alpha_n)/\big(\sqrt{\pi}c_\eta\big) \\
&\quad + (\alpha_n/2)\sqrt{T_W/T_\infty}\Big]\exp\big(-c_\eta^2\big) + \Big[(2-\alpha_n)\big(c_\eta^2 + \tfrac{1}{2}\big) \\
&\quad + \alpha_n/2\sqrt{\pi T_W/T_\infty}c_\eta\Big][1 + erf(c_\eta)]\Big\} \\
C_{t,\mathrm{FM}}(\eta; M) &= \alpha_\upsilon \cos\eta\chi(c_\eta)/\sqrt{\pi}c \\
C_{h,\mathrm{FM}}(\eta; M) &= 1/\big(2\sqrt{\pi}c^3\big)\Big\{\Big[\alpha_e\big(c^2 + \tfrac{5}{2} - 2T_W/T_\infty\big) \\
&\quad + \alpha_e^B(5-3\gamma)/(2\gamma-2)(1 - T_W/T_\infty)\Big]\chi(c_\eta) \\
&\quad - \tfrac{1}{2}\alpha_e\exp\big(-c_\eta^2\big)\Big\} \\
\chi(x) &= \exp(-x^2) + \sqrt{\pi}x[1 + erf(x)]
\end{aligned}
\tag{A1}
$$

where $c = M\sqrt{(\gamma/2)}$, γ is the ratio of specific heats, $c_\eta = c\sin\eta$, and η is the local angle of attack. Furthermore, T_W/T_∞ is the ratio of the wall temperature to the freestream temperature. In this context α_n, α_υ, α_e, α_e^B are suitable coefficients identified through experimental data.[9]

Appendix B: Inverse Problem Solution

Let us consider the following differential system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}), \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0, \qquad \boldsymbol{y} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}) \tag{B1}$$

where $\boldsymbol{x} \in \Re^n$ is the vector of state, $\boldsymbol{u} \in \Re^{n_c}$ is the vector of controls, and $\boldsymbol{y} \in \Re^{n_s}$ is the vector of desired outputs. In particular, the state vector that appears in the equations of motion is $\boldsymbol{x} = (V, \alpha, \beta, \chi, \theta, \psi, p, q, r, N, E, m)$ and the control vector is either $\boldsymbol{u} = (\delta_T, \delta_e, \delta_a, \delta_r)$, when the thrust level is modulated, or $\boldsymbol{u} = (\nu_T, \delta_e, \delta_a, \delta_r)$, when the frequency of burning is modulated. The inverse problem consists of finding the input $\boldsymbol{u}$ for an assigned output $\boldsymbol{y}$. If the problem is solved by time discretization, the numerical scheme can be written in the form

$$\boldsymbol{x}_k = \boldsymbol{h}(\boldsymbol{x}_{k-1}, \boldsymbol{u}_k), \qquad \boldsymbol{x}_1 = \boldsymbol{x}_0, \qquad \boldsymbol{y}_{k_{\mathrm{DES}}} = \boldsymbol{g}(\boldsymbol{x}_k, \boldsymbol{u}_k) \tag{B2}$$

where k is relative to the kth time step, so that $\boldsymbol{y}_{k_{\mathrm{DES}}}$ is the desired output of the discretized unknown $\boldsymbol{u}_k$ input. Introducing the expression of $\boldsymbol{x}_k$ in $\boldsymbol{y}_{k_{\mathrm{DES}}}$ gives

$$\boldsymbol{y}_{k_{\mathrm{DES}}} = \boldsymbol{g}[\boldsymbol{h}(\mathbf{x}_{k-1}, \boldsymbol{u}_k), \boldsymbol{u}_k] \tag{B3}$$

Under the hypothesis of regularity of the function $\boldsymbol{g}$, when the problem is nominal, $n_c = n_s$, and the system gives the discretized time history of the control actions necessary to obtain $\boldsymbol{y}_{\mathrm{DES}}$. If the problem is redundant and admits $\infty^{n_c - n_s}$ solutions, then a convenient solution can be obtained by assigning a scalar function $J(\boldsymbol{x}, \boldsymbol{u})$ to be minimized at each time step. The problem formulation then becomes

$$J(\boldsymbol{x}_k, \boldsymbol{u}_k) + \boldsymbol{\mu}\cdot\big\{\boldsymbol{y}_{k_{\mathrm{DES}}} - \boldsymbol{g}[\boldsymbol{h}(\boldsymbol{x}_{k-1}, \boldsymbol{u}_k), \boldsymbol{u}_k]\big\} = \min \tag{B4}$$

where $\boldsymbol{\mu} \in \Re^{n_s}$ is the vector of the Lagrange multipliers.

Appendix C: Motion Equations

Here the equations of motion of the rigid spacecraft, which are used to solve the inverse problems, are reported. The force equations for a rigid spacecraft are

$$
\begin{aligned}
T + F_X - mg\sin\theta &= m\big[\dot{u} + \big(q_B^E + q\big)w - \big(r_B^E + r\big)v\big] \\
F_Y - mg\cos\theta\sin\chi &= m\big[\dot{v} + \big(r_B^E + r\big)u - \big(p_B^E + p\big)w\big] \\
F_Z - mg\cos\theta\cos\chi &= m\big[\dot{w} + \big(p_B^E + p\big)v - \big(p_B^E + q\big)u\big]
\end{aligned}
\tag{C1}
$$

where F_X, F_Y, F_Z are the body axes components of the aerodynamic force and p_B^E, q_B^E, r_B^E are the components of Earth angular velocity in body axes

$$\begin{Bmatrix} p_B^E \\ q_B^E \\ r_B^E \end{Bmatrix} = L_{\mathrm{BV}}\begin{Bmatrix} \cos N \\ 0 \\ -\sin N \end{Bmatrix}\omega \tag{C2}$$

with

$$L_{\mathrm{BV}} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\chi\sin\theta\cos\psi - \cos\chi\sin\psi & \sin\chi\sin\theta\sin\psi + \cos\chi\cos\psi & \sin\chi\cos\theta \\ \cos\chi\sin\theta\cos\psi + \sin\chi\sin\psi & \cos\chi\sin\theta\sin\psi - \sin\chi\cos\psi & \cos\chi\cos\theta \end{bmatrix} \tag{C3}$$

Hence the dynamic and kinematic equations in state form are written as

$$\dot{V} = \frac{u\dot{u} + v\dot{v} + w\dot{w}}{V}, \qquad \dot{\alpha} = \frac{u\dot{w} - w\dot{u}}{u^2 + w^2}, \qquad \dot{\beta} = \frac{V\dot{v} - v\dot{V}}{V^2 \cos\beta}$$

$$\dot{\chi} = P + \sin\chi \tan\theta Q + \cos\chi \tan\theta R, \qquad \dot{\theta} = \cos\chi Q - \sin\chi R$$

$$\dot{\psi} = \sin\chi \sec\theta Q + \cos\chi \sec\theta R, \qquad \dot{p} = \frac{(I_y - I_z)qr - M_x}{I_x}$$

$$\dot{q} = \frac{(I_z - I_x)rp - M_y}{I_y}, \qquad \dot{r} = \frac{(I_x - I_y)pq - M_z}{I_z} \tag{C4}$$

the navigation equations are

$$\begin{Bmatrix} \dot{N}R \\ \dot{E}R\cos N \\ -\dot{R} \end{Bmatrix} = L_{\mathrm{VB}} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \tag{C5}$$

and finally the mass-consumption equation is

$$\dot{m} = -T/c \tag{C6}$$

where M_x, M_y, M_z are the body axes components of the external moment with respect to the c.g.; I_x, I_y, I_z are the principal inertia moments of the spacecraft; c is the effective exhaust velocity of the nozzle flow; L_{VB} is the inverse of L_{BV}; and $\dot{u}$, $\dot{v}$, $\dot{w}$ are obtained from the force equations. P, Q, R are the components of the relative angular velocity in body axes given by

$$\begin{Bmatrix} P \\ Q \\ R \end{Bmatrix} = \begin{Bmatrix} p \\ q \\ r \end{Bmatrix} - L_{\mathrm{BV}} \begin{Bmatrix} (\omega + \dot{E})\cos N \\ -\dot{N} \\ -(\omega + \dot{E})\sin N \end{Bmatrix} \tag{C7}$$

When the optimal c.g. trajectory is calculated, then the simplified equations are

$$\begin{aligned} \dot{V} &= F_T/m - g\sin\gamma + \omega^2 R\cos N(\sin\gamma\cos N \\ &\quad - \cos\gamma\sin\psi_v\sin N) \\ V\dot{\gamma} &= F_N/m\cos\sigma - g\cos\gamma + V^2/R\cos\gamma \\ &\quad + 2\omega V\cos\psi_v\cos N + \omega^2 R\cos N(\cos\gamma\cos N \\ &\quad + \sin\gamma\sin\psi_v\sin N) \\ V\dot{\psi}_v &= F_N/m\sin\sigma/\cos\gamma - V^2/R\cos\gamma\cos\psi_v\tan\psi_v \\ &\quad + 2\omega V(\tan\gamma\sin\psi_v\cos\psi_v - \sin\psi_v) \\ &\quad - \omega^2 R/\cos\gamma\cos\psi_v\sin N\cos N \\ \dot{R} &= V\sin\gamma, \qquad \dot{E} = V\cos\gamma\cos\psi_v/R\cos N \\ \dot{N} &= V\cos\gamma\sin\psi_v/R \end{aligned} \tag{C8}$$

where $F_T = T\cos\alpha - D$, $F_N = T\sin\alpha + L$. To the preceding system, the equation of fuel consumption is added.

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